THEORY AND APPLICATION OF COIL MAGNETIZATION

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The effective permeability of a part placed in a magnetic field differs from the permeability of the material from which the part is made. The magnetizing field required to induce a given magnetic flux density in a part is inversely proportional to the effective permeability of the part. This paper gives the results of theoretical and experimental investigations of the effective permeability of parts usually magnetized by means of coils and of the ampere turns required to obtain a flux density in the part which is suitable for magnetic particle inspection.

INTRODUCTION

When a part is placed in the magnetic field of a coil, magnetic poles appear near the surfaces at which the field enters and leaves the part(1,2,3). These poles produce, in the part, another magnetic field which is opposite in direction to the applied field and consequently they weaken the field in the part. The weakening or self-demagnetizing effect depends on the distance between the induced poles and on their concentration. Thus the self-demagnetization depends on the geometric shape of the part. Obviously, the effect also depends on the orientation of the part with respect to the applied field since the distance between the induced poles will depend on whether the long or short axis of the part is held parallel to the applied field.

In magnetic particle inspection, most parts magnetized in a coil are placed with their long axes parallel to the applied field. Hence in this paper, it is assumed that this condition is fulfilled.

EFFECTIVE PERMEABILITY

The reduction of the field inside the part is proportional to the intensity of magnetization inside(4) (also called the magnetic moment per unit volume). That is:

$$\Delta H = NI$$

where $\Delta H$ is the reduction in field strength, $N$ is the constant of proportionality, and $I$ is the intensity of magnetization. If $H_a$ is the applied field, then the field inside is:

$$H = H_a - \Delta H = H_a - NI.$$ (1)

The magnetic flux density in the part is given by:

$$B = H + 4\pi I,$$

and solving this equation for $I$ and substituting into Equation 1 gives:

$$H = H_a - \frac{4\pi}{N} (B - H).$$ (2)

Now if $\mu$ is the material permeability then $B = \mu H$ and Equation 2 can be written:

$$\frac{B}{\mu} = H_a - \frac{N}{4\pi} B (1 - \frac{1}{\mu}).$$ (3)

Solving Equation 3 for $H_a$ gives:

$$H_a = B \left( \frac{1}{\mu} + \frac{N}{4\pi} (1 - \frac{1}{\mu}) \right)$$

$$= B \left( 1 + \frac{N}{4\pi} (\mu - 1) \right).$$ (4)

If we now define the effective permeability ($\mu_{eff}$) in terms of the magnetic flux density ($B$) produced in the part by the applied field ($H_a$), to be:

$$\mu_{eff} = \frac{B}{H_a},$$

and apply the definition to Equation 4, we obtain:

$$\mu_{eff} = \frac{B}{H_a} = 1 + \frac{N}{4\pi} (\mu - 1).$$ (5)

Equation 5 shows that the effective permeability depends on the permeability of the material ($\mu$) and on a factor $N/4\pi$ which is called the demagnetizing factor. The demagnetizing factor is a function of the length-to-diameter ratio of the part. It can be calculated exactly for parts having the shapes of ellipsoids of revolution(5) but must be measured experimentally for other shapes. Bozorth(6) gives a table of demagnetizing factors for rods having various length: diameter (1/d) ratios.

Using the values of the demagnetizing factors taken from Bozorth, the effective permeabilities have been calculated, using Equation 5, for rods having various 1/d ratios and various material permeabilities. The results are shown in Table I.

A great many parts which are inspected by means of coil magnetization have 1/d ratios which are less than 15. The material permeability of common steels is greater than 500(7). E-

<table>
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<th>200</th>
<th>500</th>
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*Presented before The Society at its 14th Annual Convention, Hotel Morrison, Chicago, November 4, 1934.
References at end.
amination of Table 1 shows that when these two conditions are met, the effective permeability is very nearly independent of the material permeability and becomes a function of the $1/d$ ratio alone, e.g. for $1/d = 5$, the effective permeability is approximately 25 regardless of the material permeability. Throughout the rest of this paper, it will be assumed that the parts have $1/d$ ratios which are less than 15 and have material permeabilities greater than 500.

Sample parts were made of cold-rolled steel from stock having 0.5, 1.0, and 1.5 in. diameters. Data were taken and $H_B$ curves drawn. Figure 1 is an example of such a curve. The effective permeability of the part was taken to be the slope of the straight portion of the curve. The results are given in Table 2 and are compared there with a value calculated from the equation:

$$\mu_{eff} = 6 \left(\frac{1}{d}\right) - 5. \quad (6)$$

![Figure 1—Typical B Versus H Curve.](image)

Equation 6 is merely an approximate formula for obtaining the effective permeability. Table 2 shows that Equation 6 is sufficiently accurate for use in magnetic particle inspection. The agreement between the measured values and those given in Table 1 is also very good.

The measurements shown in Table 2 were taken at the center of the rod with the rod centered in the magnetizing coil. Measurements were also made at 1 in. intervals along the rod in order to find out whether or not $\mu_{eff}$ is constant along the rod. Table 3 shows the results of these measurements. The results are symmetrical about the center of the rod; i.e., the effective permeability is the same at a given distance on each side of the center of the rod, consequently the table gives data for only one side. It is apparent that the effective permeability is constant over most of the length. The induced poles are located near the ends of the rod and their presence makes measurement of flux density near the ends very difficult. The apparent drop in effective permeability near the ends is more likely to be due to errors in measurement of $B$ than to an actual drop in permeability.

So far, it has been shown that for many parts which are magnetized in a coil, the effective permeability is dependent only on the $1/d$ ratio and is constant along its length. An approximate formula for $\mu_{eff}$ has been given and shown to hold for $1/d$ ratios less than 15 and material permeabilities greater than 500. Now it will be shown that the effective permeability can be used as an indicator of the number of ampere-turns required to produce a flux density, in a part, which is suitable for magnetic particle inspection.

### AMPERE- Turns Required FOR MAGNETIC PARTICLE INSPECTION

The effective permeability was defined by the equation:

$$\mu_{eff} = \frac{B}{H_0},$$

where $B$ is the flux density in the sample produced by the applied field $H_0$. Solving this equation for $H_0$ gives:

$$H_0 = \frac{B}{\mu_{eff}}. \quad (7)$$

Equation 6 gives $\mu_{eff}$ as a function of $1/d$. Using that relation, Equation 7 becomes:

$$H_0 = \frac{B}{6(1/d) - 5}. \quad (8)$$

If the flux density ($B$) required to show a given defect is known, then Equation 8 allows the field strength which will produce that flux density in the part to be calculated. The field strength is directly proportional to the product of the number of turns in the coil and the current flowing in the coil so that Equation 8 can be written:

$$nI = \frac{kB}{6(1/d) - 5}. \quad (9)$$

where $nI$ is the number of ampere-turns and $k$ is the proportionality constant. The value of the constant $k$ will depend on the dimensions of the magnetizing coil and on the position of the test sample in the coil. The constant $k$ also contains conversion constants so that the equation will balance dimensionally (see Section on Units).

The value of $B$ in Equation 9 will depend on the defect to be found; that is, on its size, shape, location, etc. Experiments have shown that $B$ can vary between 200,000 lines per in.$^2$ and 160,000 lines per in.$^2$ and that most defects can be located if $B$ has the value 70,000 lines per in.$^2$. In fact, the rule-of-thumb generally used to determine the amount of current re-
quired for circular magnetization (1000 amp per in. sample diameter) produces a flux density of about 70,000 lines per in.\(^2\) in most steels.

To obtain a workable thumb-rule for use in coil magnetization, B was chosen as 70,000 lines per in.\(^2\) and the constant in Equation 9 was evaluated experimentally for three different coil sizes. It was found that, with sufficient accuracy, Equation 9 can be replaced by:

\[
 nI = \frac{45,000}{1/d} \text{ampere-turns (10)}
\]

providing that the part is located at the bottom of the coil and that its cross-sectional area is not greater than one-tenth that of the magnetizing coil. (See section on Magnetizing Coil.)

Units

In the plane of a circular coil, the magnetic field varies from a minimum value at the geometric center to a maximum value at the windings. When adjusted for units, the equation giving the field strength at any point in the plane of the coil is given by Reddick and Miller\(^8\) to be:

\[
 H_p = \frac{2\pi NI}{25.4 R} \left( \frac{E(m)}{1 - m^2} \left( \frac{\pi}{2} \right) \right) \tag{11}
\]

where \( H_p \) is the field strength in oersteds at the point,

- \( P \) on the radius of the coil,
- \( NI \) is the ampere-turns in the coil,
- \( m \) is the ratio \( S/R \),
- \( S \) is the distance from \( P \) to the center of the coil,
- \( R \) is the radius of the coil in inches, and
- \( E(m) \) is the elliptic integral of the second kind of argument \( m \).

According to Equation 11, the field strength applied to a part placed in a circular coil depends on the ampere-turns, \( NI \), the coil radius, \( R \), and on the placement of the part in the coil, \( m \). For a given coil and for a particular placement of the part, the quantities \( E(m) \), \( R \), and \( m \) are constants and Equation 11 becomes

\[
 H_p = \frac{NI}{k}, \tag{12}
\]

where \( H_p \) has been changed to \( H_p \) to indicate the applied field under these conditions. Substitution of Equation 12 into Equation 8 leads directly to Equation 9.

Magnetizing Coil

If a magnetizing coil is to be replaced by one having a different diameter and if the field strength applied to a part is to remain the same, then the ampere-turns required may depend on the diameter-ratio of the two coils. Proper placement of the part in the coil can remove the diameter dependence.

If the part is centered in the coil then in Equation 11, \( S \) is zero so that \( m \) is zero and \( E(m) \) is \( \pi/2 \). Equation 11 then becomes:

\[
 H_p = \frac{2\pi NI}{25.4 R}. \tag{13}
\]

Indicating the two coils by subscripts, the ratio of the field strengths at the center is, by Equation 15:

\[
 \frac{H_1}{H_2} = \frac{(NI)_1}{(NI)_2} \cdot \frac{R_2}{R_1}. \tag{14}
\]

and solving for the ampere-turns ratio gives:

\[
 (NI)_1 = \frac{H_1}{H_2} \cdot \frac{R_1}{R_2} \cdot (NI)_2. \tag{15}
\]

According to Equation 15, if the field strengths are to be the same then the ratio of the ampere-turns must equal the ratio of the coil radii. If coil 2 has a diameter which is twice that of coil 1, then the ampere turns for coil 2 must be twice that for coil 1.

In magnetic particle inspection, the most common position of the part is on the bottom of the coil. For this case, \( S = R - r_p \) and \( m = 1 - (r_0/R) \) where \( r_0 \) is the radius of the part. Using the same notation as before and after some algebraic manipulation, Equation 11 becomes:

\[
 H_1 = \frac{(NI)_1}{E(m_1)} \cdot \left( 2 - \frac{R_2}{R_1} \right) \tag{16}
\]

Now the function \( E(m) \) is a relatively slow varying function so that the ratio of the field strength depends on the coil radii mostly thru the factors of the form \( 2 - r_p/R \). If \( r_p/R \) is small compared to 2, then the ratio of field strengths is very nearly independent of the coil diameters. Numerical calculation shows that for \( r_p = 0.5 \) in., \( R_1 = 6 \) in., and \( R_2 = 12.5 \) in., Equation 16 gives:

\[
 \frac{H_1}{H_2} = 1.09 \cdot \frac{(NI)_1}{(NI)_2}. \tag{17}
\]

According to Equation 17, if a 1.0 in. diameter part is placed at the bottom of a 12 or 25 in. diameter coil, it will experience the same magnetizing force (within 9 per cent), if the ampere-turns are the same for the two coils. Calculations show that if the area of cross-section of the part is not greater than one-tenth the area of the coil, then the ampere-turns required to produce a given field strength can be considered to be independent of the coil diameter providing that the part is placed at the bottom of the coil. The thumb-rule given by Equation 10 depends on this fact.

**SUMMARY**

It has been shown that when a part is magnetized by placing it in the field of a coil, its effective permeability differs from the permeability of the material of which the part is made. For the range of parts considered, the effective permeability is independent of the material permeability and depends only on the length-to-diameter ratio. For cylindrical parts, the effective permeability is given with sufficient accuracy by:

\[
 \mu_{eff} = 6 (1/d) - 5. \tag{6}
\]

The ampere-turns required for magnetic particle inspection is given approximately by:

\[
 nI = \frac{45,000}{1/d} \tag{10}
\]

which is based on the assumption that a flux density of 70,000 lines per in.\(^2\) will show most defects and that the part is placed at the bottom of the coil.

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**REFERENCES**

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6. Bozorth, op. cit. p 849
7. Kraus, op. cit. p 208